Chifley College Dunheved Campus

Learn Discover Inspire

STAGE 5
MATHEMATICS
HOMEWORK BOOK

Linear Calculus Proofs
Triangles Addition Equations Decimals
Division Parabolas Fractions
Quadratic Counting
Percentages Denominator
Trig Expressions
MATHEMATICS Multiplication
Subtraction Logarithms Logic
Numbers Numerator Congruent
Theories Money Algebra
Math Teacher Graphs
Stats Formulas Roots
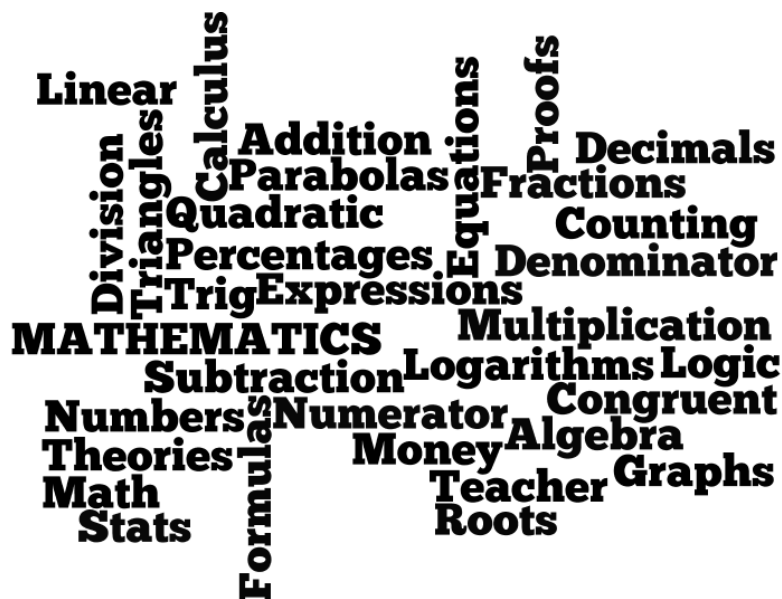
Whole Numbers B

Another word that can be used instead of 'whole number' is 'integer'.
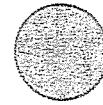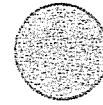
Order of Operations: BODMAS

Arithmetic operations are carried out in a specific order.

B **Brackets:**   Brackets come first. Calculations within the brackets are carried out first.
O **Of:**   'Of' is converted into Multiply (i.e. 'of' means '×').
D **Division:**   Division and Multiplication take place next.
M **Multiplication:**
A **Addition:**   Addition and Subtraction take place last.
S **Subtraction:**

Example	Answer	
Evaluate.	$2 + 4 \times (17 - 2) \div 5$	$17 - 2 = 15$ [B]
$2 + 4 \times (17 - 2) \div 5$	$= 2 + 4 \times 15 \div 5$	$4 \times 15 \div 5 = 12$ [M/D]
	$= 2 + 12$	
	$= 14$	

Division can be represented by the symbol $\div$ or the 'dividing line', so $24 \div 8$ is the same as $\frac{24}{8}$.
'Of' means multiply, so $\frac{1}{8}$ of 24 means $1 \div 8 \times 24$.

practice

Evaluate.

1	$362 + 109$	**2**	$2235 - 1047$
3	452×8	**4**	$275 \div 5$
5	$3 \times (80 + 35) - 45 \times 3 + 63$	**6**	$\frac{3}{8}$ of $32 - 5 \times 2 + (52 - 13 \times 4) \times 2$

Evaluate.

7 125 + 657	**8** 3655 − 2996
9 $2 \times (45 - 6 \times 7 + 8) + 15$	**10** $34 \div (5 \times 3 - 13) - 17$
11 $[\frac{2}{5}$ of $80 + 2 \times 4] \div 8$	**12** 404 + 44044 − 44 + 4
13 $4 \times (\frac{3}{7}$ of $21 + 6) \div 5 + 167$	**14** $14 + \frac{7}{8}$ of $48 \times 3 - 125$
15* $5875 \div 25 + 12 \times 13$	**16** $21 - \frac{2}{7}$ of $(2 \times 18 - 16 \div 2)$
17 $(16 \times \frac{1}{2}$ of $18 - 44) \div 5$	**18** A tennis team consists of six players. Half the teams have an emergency player. If there are thirty-four teams in the tournament, how many tennis players are there?
19 $35 + 4 \times (\frac{1}{2}$ of $(\frac{7}{8}$ of $64)) \div 7 + 14$	

8

Big Factor Trees

Have I got all the factors?
A factor tree is one way of helping you
to answer 'yes' to this question.

All whole numbers can be written as the product of a set of smaller whole numbers.
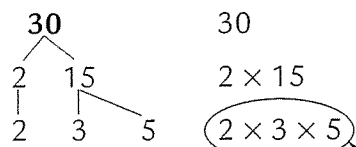
For example, $15 = 3 \times 5$ and $30 = 2 \times 3 \times 5$.

Some numbers can only be written as the product of themselves and one. These special numbers are called '**prime**' numbers. Note that one itself is not a prime number. All other numbers are called '**composite**' numbers.
The first ten primes are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.
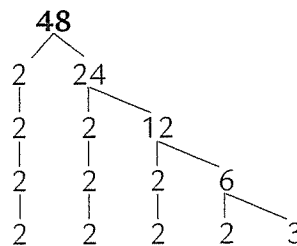
What is the next prime number? [   ]

A factor tree is a pictorial representation of the set of factors that make up a number. It is found by dividing the prime numbers (in order) into the factors.

So factor pairs of 30 are:
30×1
15×2
10×3
6×5

and factor pairs of 48 are:
48×1
24×2
16×3
12×4
8×6

Thus 30 is divisible by:
1, 2, 3, 5, 6, 10, 15 and 30.

and 48 is divisible by:
1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

practice

1	Why is there only one *even* prime number and what is it?		

2 Create a factor tree for 18.	**3** What are the prime factors of 18?	**4** State the numbers by which 18 is divisible.

Create factor trees for the following numbers and use them to find all the factor pairs of each number.

5 21	**6** 17	**7** 36
8 40	**9** 56	**10** 32
11 100	**12** 81	**13** 39
14 128	**15** 72	**16** 96

17   State the next 10 prime numbers after 29.

Rounding Decimals

Quoting an answer to the correct
(or appropriate) number of
decimal places is important.

When rounding a decimal number to a specific number of decimal places, look at the value of the next decimal place. Increase the last decimal place by 1 if this value is 5 or greater.

Example 1	Answer 1
Round 13.446237 to two decimal places (dp).	13.44⑥237 (look at 3rd dp). = 13.45 6 > 5 hence add 1

Example 2	Answer 2
Round 2.882359 to three decimal places	2.882③59 (look at 4th dp) = 2.882 3 < 5 so leave as is

Only round the answers and always work to at least two more decimal places than is required in the answer, applying BODMAS to all calculations.

For example, the **answer** in Example 1 is 13.45, but any *further calculations* must use 13.446237, not 13.45.

Rounding to the nearest whole number is the same as rounding to 0 (zero) decimal places.

practice

Round the decimal values below to the number of decimal places (dp) specified in the brackets.

1 834.5548 (2 dp)	2 1.9827 (3 dp)
3 45.537 (nearest whole number)	4 9.27557 (2 dp)
5 63.546365 (2 dp)	6 2.00988 (1 dp)
7 0.001832 (4 dp)	8 4.667872 (2 dp)
9 0.342991 (3 dp)	10 26.99978 (3 dp)

Round the decimal values below to the number of decimal places (dp) specified in the brackets.

11 6.2236 (1 dp)	**12** 356.89 (nearest whole number)
13 9.6345 (0 dp)	**14** 3.44538 (2 dp)
15 0.787878 (3 dp)	**16** 0.443322 (1 dp)
17 6.719902 (4 dp)	**18** 98.29922 (2 dp)
19 26.7223 (0 dp)	**20** 14.31992 (3 dp)
21 7.78228 (3 dp)	**22** 200.0021 (1 dp)
23 25.99891 (2 dp)	**24** 565.899 (2 dp)
25 1.23050 (2 dp)	**26** 19.9973 (2 dp)

Fractions B

Fractions, decimals and percentages are all related. Consider $\frac{1}{2}$, 0.5 and 50% mean the same thing.

Identify the correct answer (from box A–D).

		(A)	(B)	(C)	(D)
1	The correct way to write a fraction is	12\13	12/13	$\frac{12}{13}$	12|13
2	The two equivalent fractions are				
3	$\frac{1}{3}$ is equivalent to	$\frac{2}{3}$	$\frac{12}{24}$	$\frac{4}{6}$	$\frac{5}{15}$
4	The improper fraction is	$\frac{5}{6}$	$\frac{31}{32}$	$\frac{57}{56}$	$\frac{10}{30}$
5	The Lowest Common Multiple (LCM) of 6 and 8 is	2	12	24	48
6	$\frac{1}{2} + \frac{1}{4}$ equals	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{2}{6}$
7	'It is no good', said Mum. 'A quarter of the apples I bought are rotten.' If Mum bought twenty apples, how many were rotten?	80	5	15	20
8	Jan has to give half her pocket money to her twin brother Jon. If Jan is given $15.00, how much does she give to Jon?	nothing	$5.00	$7.00	$7.50
9	A third of the class has got the flu. If there are twenty-one people in the class, how many of them do *not* have the flu?	7	14	21	63

10 Colour in $\frac{3}{4}$ of this circle.

11 Arrange these fractions in ascending order (i.e. from smallest to largest).

$$\frac{3}{7} \qquad \frac{5}{21} \qquad \frac{13}{21} \qquad \frac{2}{3}$$

12 Write $\frac{24}{72}$ in its simplest form.

13 Convert the improper fraction $\frac{45}{8}$ into a mixed number.

14 Convert the mixed number $12\frac{2}{3}$ into an improper fraction.

15 Calculate $2\frac{1}{3} + 5\frac{3}{4}$.

16 Calculate $\frac{3}{4} \times \frac{8}{9}$.

17 Calculate $\frac{25}{6} \div \frac{50}{7}$.

18 In a school of 1200 students, $\frac{5}{8}$ travel by public transport, $\frac{1}{3}$ by car, and the rest travel by bicycle or walk.

(a) How many students travel by public transport?

(b) How many students travel by car?

(c) How many students travel by bicycle or walk?

(d) What fraction of the total number of students is the group who travel by bicycle or walk?

Magic Squares

Unit 1.10

A Magic Square is a set of numbers in a square grid where each row, column and diagonal add to the same value, called the 'Magic Number'.

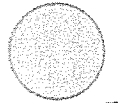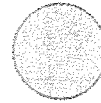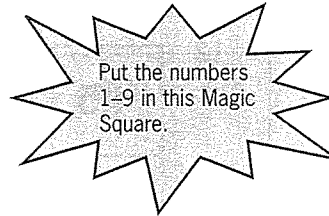| 5	15	13
19	11	3
9	7	17

Here is a 3 × 3 square.

Each row, column and diagonal add to 33, which is the Magic Number.

Middle Number = $\dfrac{\text{Magic Number}}{3}$

Put the numbers 1–9 in this Magic Square.

practice

Complete the following Magic Squares.

1

21	7	
11		
13		

Magic Number ....

2

13	43	37
	31	

Magic Number ....

3

28	24	20
		27

Magic Number ....

4

4.2		
2.2		3.8
2.6		

Magic Number ....

5

1.0	3.0	2.6
	1.4	

Magic Number ....

6

1.0		
	2.8	0.4
		4.6

Magic Number ....

7 List the numbers in a Magic Square in ascending order. Can you see a pattern?

Complete the following Magic Squares.

8

19.5	12	13.5
		10.5

Magic Number ....

9

45	70	35
	50	
	30	

Magic Number ....

10

13		
6		14
11		

Magic Number ....

11

$1\frac{1}{4}$		$4\frac{1}{4}$
		$5\frac{3}{4}$

Magic Number ....

12

4.5	4	
	5	

Magic Number ....

13

$4\frac{1}{2}$	7	$3\frac{1}{2}$
	5	

Magic Number ....

14 List the numbers in each Magic Square. What do you notice?

15 Use this information to create three of your own 3 × 3 Magic Squares.

Magic Number ....

Magic Number ....

Magic Number ....

Unit 1.R

Number Revision

How many apples a day keep the doctor away?
How many jellybeans make a score?

Name: _____

Class: _____ Due date: _____

| Content | Working out | Setting out |
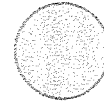
Circle the correct answer (A–D).

1	What is the largest number in the following list? 5 15 35 0 75 11 36 12 77 78 81 49	(A) 75 (B) 78 (C) 0 (D) 81	

1 What is the largest number in the following list?

5 15 35 0 75 11
36 12 77 78 81 49

(A) 75 (B) 78
(C) 0 (D) 81

2 What pair of numbers from the following list are the same?

75% $\frac{4}{3}$ 75 50 0.5 5
0.75 $\frac{1}{5}$ 25% 1 0.6 1.3

(A) 0.5, 50 (B) 75%, $\frac{4}{3}$
(C) 0.75, 75% (D) $\frac{1}{3}$, 5

3 What is the smallest number in the following list?

1.0100 1.1 1.1101
1.011 1.001 1.01

(A) 1.1 (B) 1.001
(C) 1.0100 (D) 1.011

4 $(3 + 4) \times 5 - 4$ equals

(A) 15 (B) 31
(C) 7 (D) 16

5 $(5 + 15) \div 5$ equals

(A) 3 (B) 4
(C) 8 (D) 16

6 Which of the following numbers is greater than 6.05?

(A) 6.049 (B) 6.02
(C) 6.2 (D) 5.06

7 The next even number after 35 is

(A) 36 (B) 37
(C) 38 (D) 40

8 $\frac{1}{2} + \frac{2}{3}$ equals

(A) $1\frac{1}{6}$ (B) 1
(C) $\frac{4}{3}$ (D) $\frac{5}{6}$

9 Find the set of all factor pairs for 72.	
10 Arrange the following numbers from smallest to largest. 1110 1011 1010 1101 1001 1100 1111	
11 Are the following statements True (T) or False (F)? (a) $\frac{4}{3}$ equals 1.3 (b) 10.1021 is greater than 10.1201	(a) (b)
12 Find five consecutive (a) integers, the largest being 16 (b) odd integers, the largest being 13.	(a) (b)
13 Evaluate, correct to 2 decimal places. (a) $3.564 \times 4.6 \times (13.453 - 6.81)$ (b) $\dfrac{12.332 \times 9.001}{(3.4 + 2.345)}$	(a) (b)
14* Evaluate, correct to 2 decimal places. (a) $13.95 \times (5.544 + 1.88) \times (24.553 - 17.88)$ (b) $\dfrac{13.1665 \times 9.721}{2 \times (5.4 + 5.815)}$	(a) (b)

15 One night in July the weather report showed the following maximum temperatures (°C):

Melbourne	14	Ballarat	10
Mildura	16	Lakes Entrance	15
Mt Hotham	2	Bendigo	8

(a) Which place recorded the

(i) maximum temperature?

(ii) minimum temperature?

(b) What was the difference between these two temperatures?

(c) In summer the temperatures are all 10 °C higher. What is the difference between the maximum and minimum temperatures now?

(d) Arrange the places from warmest to coldest.

Unit 2.1 Mental Challenge A

Most of us think in 'straight lines'. Light travels in straight lines. But now, with fibre optics, light can travel around corners. See if you can think 'around corners'.
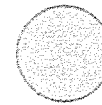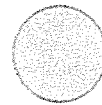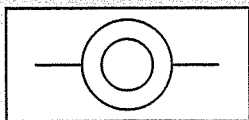
Example 1

What does the diagram in the box represent?

Answer 1

A Mexican wearing a sombrero riding a bicycle (if you are looking down from above).

no calculator

Example 2

A palindromic number reads the same forwards or backwards. For example, 14 541.
How many three-digit palindromic numbers are there?

Answer 2

101, 111, 121, ... 191 (10)   606, 616, 626, ... 696 (10)
202, 212, 222, ... 292 (10)   707, 717, 727, ... 797 (10)
303, 313, 323, ... 393 (10)   808, 818, 828, ... 898 (10)
404, 414, 424, ... 494 (10)   909, 919, 929, ... 999 (10)
505, 515, 525, ... 595 (10)
There are 90 three-digit palindromic numbers.

1	How far can you walk into a forest?	
2	Slibber doubles in volume every ten minutes. A jar containing Slibber was full at 9:30 am. When was the jar half full?	
3	(a) Arrange the digits of the number 16 574 in descending order. (b) Arrange the digits of the number 16 574 in ascending order. (c) Subtract the number formed in (b) from the number formed in (a).	(a) (b) (c)
4	Mandy found fifteen mice in her farmhouse kitchen during a mouse plague. She caught all but four. How many were left?	

5 How many palindromic numbers are there between 50 and 500?

6 What does the diagram in the box represent?

7 What is the smallest number that is divisible by 2, 3, 4, 5 and 6?

8 Anne's wheelbarrow can hold up to five bluestone blocks at a time. She has a load of 6 dozen (72) delivered and left outside the front of her house. She has to carry them in her wheelbarrow to the back garden. How many trips does she make?

9 Ian has a ladder with fifteen rungs 12 cm apart and Alasdair has a ladder with eighteen rungs 11 cm apart.
(a) Whose ladder is taller?
(b) By how much?

(a)   (b)

10 What does the following represent ?
BPOUOSTSS

11 How many hours are there in a week?

12 A thousand beads are strung on a long thread. Three different colours are used (**R**ed, **W**hite and **B**lue) and the beads are strung in order R W B R W B R ....
What colour is the 100th bead?

13* A cowboy rode into town on Friday.
He stayed three days and left on Thursday.

Explain how this is so.
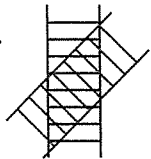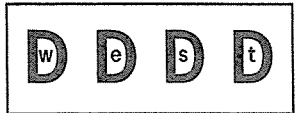
Length Conversions

When working with measurements it is important that they are in the same unit and that the appropriate units are used.

Name:_____

Class:_____ Due date:_____

| Content | Working out | Setting out |

To change from a larger unit of measurement into a smaller one, we multiply by the values shown below. To change from a smaller unit of measurement into a larger one, we divide by the values shown below.

	$\times 10$		$\times 100$		$\times 1000$	
millimetre (mm)	$\leftarrow$ $\rightarrow$ $\div 10$	centimetre (cm)	$\leftarrow$ $\rightarrow$ $\div 100$	metre (m)	$\leftarrow$ $\rightarrow$ $\div 1000$	kilometre (km)

Example 1

Express 135 000 mm in m.

Answer 1

135 000 mm = 135 000 ÷ 10 ÷ 100 m
= 135 m

Example 2

Express 3.45 m in mm.
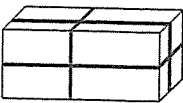
Answer 2

3.45 m = 3.45 × 100 × 10 mm
= 3450 mm

practice

Convert the following measurements into the units given in the following brackets.

1	13.4 mm (cm)		2	23 m (km)
3	88 430 cm (km)		4	2.232 m (mm)
5	44.56 km (m)		6	Is 34 m bigger than 3400 cm?
7	1.62 km (cm)		8	152 354 mm (m)

9 Express 23.56 km in m.	10 Convert 776 800 mm into km.
11 Express 1355.9 m in km.	12 Convert 0.0034 km into cm.
13 How many mm are there in 23.004 m?	14 Express 6500 mm in m.
15 Express 2.4537 km in cm.	16 Convert 13.45 cm into m.
17 Arrange in ascending order of magnitude: 12.5 cm, 12 500 mm, 1.25 m.	18 Find the perimeter (in cm) of a rectangle 12.4 cm long and 345 mm wide.
19 An ant travels 1443 cm on day 1, then 45.6 m on day 2 and 56 400 mm on day 3. How far (in m) did he travel?	20 Arrange in descending order of magnitude: 0.000342 km, 342 000 mm, 3.42 m.
21 Find (in cm) the perimeter of a hexagon whose sides measure 3.4 cm, 6 cm, 0.45 m, 1335 mm, 41 mm and 3.43 cm.	22 How much ribbon is needed to tie up this rectangular box 34 cm by 125 mm by 8.5 cm, allowing 75 cm extra for the bow?

Area Conversions

A square foot is approximately 929 cm².

Name:_____

Class:_____ Due date:_____

| Content | Working out | Setting out |

To change from a larger unit of measurement into a smaller one, we multiply by the values shown below and to change from a smaller unit of measurement into a larger one, we divide by the values shown below.

$$\begin{array}{ccccccc}
 & \times 10^2 & & \times 100^2 & & \times 1000^2 & \\
\text{millimetre}^2 & \leftarrow & \text{centimetre}^2 & \leftarrow & \text{metre}^2 & \leftarrow & \text{kilometre}^2 \\
\text{mm}^2 & \rightarrow & \text{cm}^2 & \rightarrow & \text{m}^2 & \rightarrow & \text{km}^2 \\
 & \div 10^2 & & \div 100^2 & & \div 1000^2 &
\end{array}$$

Also:
1 hectare = 10 000 m²
(ha)

Example 1

Express 135 000 mm² in cm².

Answer 1

$135\,000 \text{ mm}^2 = 135\,000 \div 10^2 \text{ cm}^2$
$= 1350 \text{ cm}^2$

Example 2
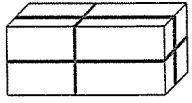
Express 0.9245 m² in mm².

Answer 2

$0.9245 \text{ m}^2 = 0.9245 \times 100^2 \times 10^2 \text{ mm}^2$
$= 924\,500 \text{ mm}^2$

practice

Convert the following measurements into the units given in the following brackets.

1 13 214 mm² (cm²)	**2** 452 300 m² (km²)
3 87 730 m² (ha)	**4** 0.0232 m² (mm²)
5 0.00456 km² (m²)	**6** Is 1.5 m² bigger than 1500 cm²?
7 1.62 km² (ha)	**8** 15 235 400 mm² (m²)

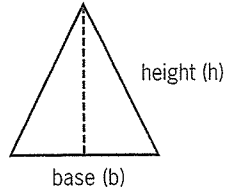|	
9 Express 4.56 km^2 in ha.	**10** Convert 745 800 mm^2 into m^2.
11 Express 1342.2 ha in m^2.	**12** Convert 0.000036 km^2 into cm^2.
13 How many cm^2 are there in 25.004 m^2?	**14** Express 9500 m^2 in ha.
15 Express 62.437 km^2 in m^2.	**16** Convert 11.45 cm^2 into m^2.
17 Arrange in ascending order of magnitude: 1250 cm^2, 1 250 000 mm^2, 12.5 m^2.	**18** Find the area (in cm^2) of a rectangle 22.4 cm long and 135 mm wide.
19 Two adjoining blocks of land are sold to the same buyer. The blocks are 2400 m^2 and 3540 m^2. What (in ha) is their combined size?	**20** Arrange in descending order of magnitude: 0.0000142 km^2, 1.42 m^2, 142 000 cm^2.
21 Find (in cm^2) the area of a triangle whose base is 2335 mm and height is 0.023 m.	**22** How much paper is needed to wrap up this rectangular box 0.24 m by 95 mm by 10.5 cm?

Area Formulas

A square yard is made up of 9 square feet.
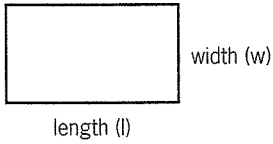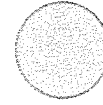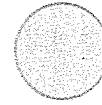
Rectangle
Area, A = l × w

Triangle
Area = $\frac{1}{2}$b × h
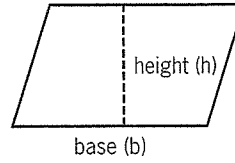
Parallelogram
Area = b × h

Square
Area = l × l
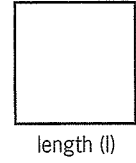
height (h) — base (b) — width (w) — length (l) — base (b) — length (l)

Example 1

Find the area of the square illustrated.
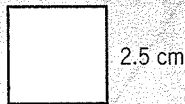
2.5 cm

Example 2

Determine the area of the rectangle illustrated.

3.4 cm
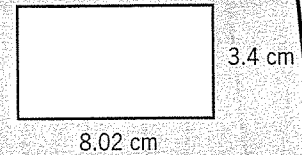
8.02 cm

Answer 1

All the sides are equal,
thus area = 2.5 × 2.5
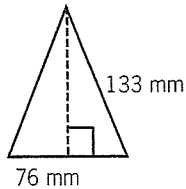= 6.25 cm²

Answer 2

Length = 8.02 cm,
thus area = 8.02 × 3.4
= 27.268 cm²

Remember: The answer is 'area = value, together with a unit of measurement', not just a 'value'.
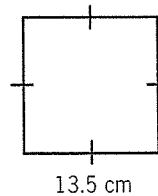
practice
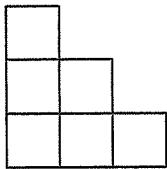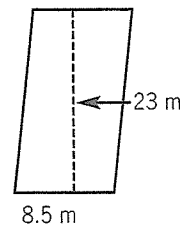
Find the area of the following shapes.

1 133 mm 76 mm	2 13.5 cm
3 Six 2 cm squares arranged as shown	4 ←23 m 8.5 m
5 A triangle, base 4.5 cm and height 25 mm	6 A rectangle 3.4 m by 1.3 m

Find the area of the following shapes.

7 A triangle of base length 34.5 cm and height 25.5 cm	**8** A square of side length 5.16 m
9 A rectangle 5.4 cm by 2.5 cm	**10** A parallelogram, base 5.3 m, height 1.2 m
11 85 mm 6.3 cm	**12** 23 m 15 m
13 Ten 4-cm squares arranged as shown 	**14** 0.12 m 11 cm
15 (a) Six 5-cm squares arranged as shown 	**15** (b) A rectangle 100 mm by 15 cm
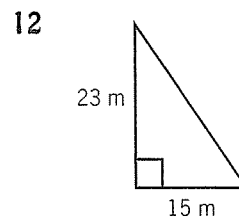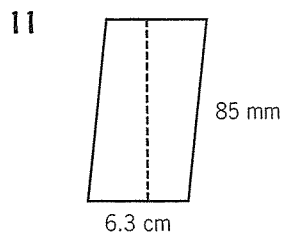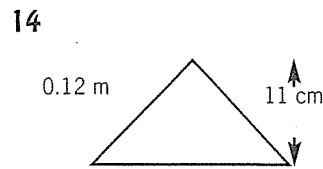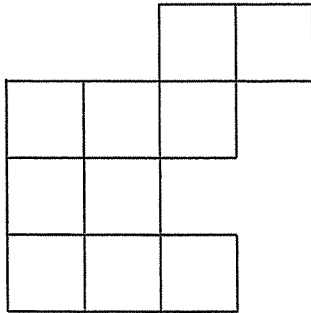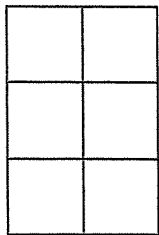
15 (c) Comment on the answers to 15 (a) and (b) above.

Unit 3.R

Measurement Revision

When is a door not a door? When it is ajar.

Name: _____

Class: _____ Due date: _____
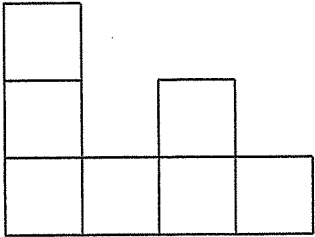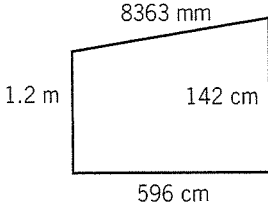
Content	Working out	Setting out
◯	◯	◯
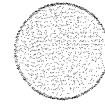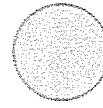
Convert the following measurements into the units shown in the brackets.

1 244 cm (m)	2 2.36 km (m)
3 1 284 300 mm (km)	4 17.889 m (mm)
5 0.0012006 km² (m²)	6 Is 3.4 m² bigger than 340 000 cm²?
7 12.645 km² (ha)	8 65 200 400 mm² (m²)

Calculate the perimeter of the following shapes.

9 Seven 3.5 cm squares arranged as shown	10 8363 mm, 1.2 m, 142 cm, 596 cm
11 A regular hexagon, side length 56 mm	12 A square 1.43 m by 1.43 m

Determine the area, correct to one decimal place, of the following shapes.

13 A triangle of base length 134.5 cm and height 87.67 cm	**14** A square of side length 5.24 mm
15 A rectangle 25.6 cm by 135 mm	**16** A parallelogram, base 403 cm, height 3.2 m

17

3.2 m   3.1 m

654 cm

18

14.3 m

0.0052 km
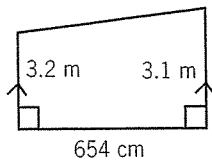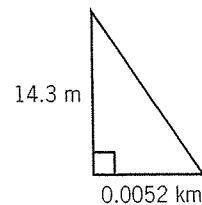
19 Nine 5.4 cm squares arranged as shown
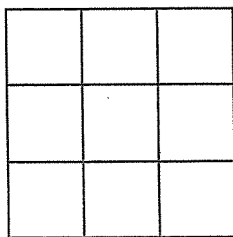
20 How many hours are there in 2 weeks and 5 days?

21 Which months have 31 days?

22 How many minutes are there between 3:35 pm and 11:24 pm on the same day?

What is Algebra?

Algebra is a language.
A pronumeral stands in place of a number.
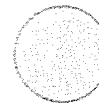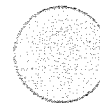
A pronumeral stands in place of a number.

Suppose I am 17 years old and my sister is 5 years younger. She is 17 – 5 years old, i.e. 12.

A pronumeral (y) can be used to represent my age, then if I am y years old and my sister is 5 years younger, she is $y - 5$ years old.

In eight years, my age will be $y + 8$ and, four years ago, my age was $y - 4$.

Double my age is $2 \times y$, which is written as $2y$.

Thus, if I am 15 now, in eight years I will be 15 + 8, i.e. 23 years old.

Pronumerals represent numbers. For example:

when $g = 2$, the value of the expression $3g + 1$ is $3 \times 2 + 1 = 7$ and

when $g = 5$, the value of the expression $3g + 1$ is $3 \times 5 + 1 = 16$.

practice

If $m = 3$ and $n = 5$, evaluate (i.e. find the value of) the following expressions.

1 $2m$	**2** $4n + 6$
3 $2m \div 3$	**4** $6mn$
5 $\dfrac{2n + 5}{m}$	**6** $\frac{1}{3}$ of $(7n + 1)$
7 $3(n + 4)$	**8** $(5n + 2) \div m$

Complete the tables by substituting into the given rule.

9 $y = 2x - 3$

x	y
-2	
-1	
0	
10	

10 $y = 2(x - 3)$

x	y
-2	
-1	
0	
10	

11 $y = 2x + 3$

x	y
-2	
0	
8	
10	

12 $y = 2(x + 3)$

x	y
-2	
-1	
6	
8	

13   Answer the following questions, given that I am y years old now:

(a)   How old will I be in 4 years?

(b)   My mother is 26 years older than me. How old is she?

(c)   My granny is five times as old as me. How old is she?

(d)   My brother Tom is 4 years younger than me. How old is he?

(e)   Mary is twice as old as me. How old is she?

(f)   Kevin is 3 years older than Mary, who is twice my age. How old is he?

(g)   Charlie, my baby brother, is 9 years younger than me. How old is he?

(h)   Mary's baby is 22 years younger than Mary. How old is the baby?

14   Suppose $y = 12$, then what are the numerical answers in question 13 (above)?

(a)   (b)   (c)   (d)   (e)   (f)   (g)   (h)

Algebraic 'Tidying up'

When working with algebraic expressions, it is important that everything is kept in its simplest form, always 'tidied up'.

To 'tidy up' algebraic expressions we look for, and collect, 'like' terms.

Consider the expression $3y + 5x - y$.

There are three terms in this expression:

The first term $(3y)$ refers to the pronumeral y, stating that there are three of them, i.e. $y + y + y$.

The last term also refers to the pronumeral y, stating that one must be subtracted (removed)

i.e. $3y - y = 2y$.

The middle term $(5x)$ refers to the *different* pronumeral x, stating that there are five of them.

So the expression $3y + 5x - y$ is 'tidied up' into $2y + 5x$.

i.e. $3y + 5x - y = 2y + 5x$ $\overset{\text{3 lots of } 2x}{}$

Remember: $3 \times 2x = 6x$ $2x + 2x + 2x = x + x + x + x + x + x$

$= 6x$

Example

Consider the expression $6g + 7d + 3d - 4g + 4g + 5d + g + 2$

(a) State the number of terms.   **(b)** State the pronumerals.

(c) Simplify ('tidy up') the expression.

Answer

(a) There are eight terms.   **(b)** d and g

(c) $6g + 7d + 3d - 4g + 4g + 5d + g + 2 = 7d + 3d + 5d + 6g - 4g + 4g + g + 2$

$= 15d + 7g + 2$

d-terms:
$7 + 3 + 5 = 15$
g-terms:
$6 - 4 + 4 + 1 = 7$

Hint: Tick each term as you 'collect' it so that you make sure no terms are left out.

practice

(a) State the number of terms in the following expressions and **(b)** simplify them by collecting all the 'like' terms.

1	$3x + 4 + 6 + 4x + 8 + 12x$		**2**	$5u - 4r + 7r + 12u$
	(a) (b)			(a) (b)
3	$24t + 57m + 23m + 45t$		**4**	$4c - 12 + 4d + 34 + 7d$
	(a) (b)			(a) (b)
5	$5h + 13 - 3h + 2 - 2h$		**6**	$24d - 5 + 16 + 12 + 2d - 26d$
	(a) (b)			(a) (b)

(a) Simplify each expression by collecting all the 'like' terms and
(b) state the number of terms in the final expression.

7 $12X + 3x - 2y + 3y + 8 + 4X - 2x$

(a)

(b)

8 $7 + 15t - 7 + 14 - 12t - 5 - 3t + 1$

(a)

(b)

9 $5d + 6b - 6h + 11b + d + 9h + 4$

(a)

(b)

10 $3 \times 5g - 5g + 14g + 3 \times 2g - g$

(a)

(b)

11 $12d - 4 \times 2s + 3 \times 6t + 24 - 16t + 1$

(a)

(b)

12 $6 \times 5w + 35 - 4 \times 7w - 6 \times 7 + 7 - 2w$

(a)

(b)

13   Mia buys 6 cans of cat food @ \$$x$ per can and 7 cans of dog food @ \$$2x$ per can. If she also buys a bottle of lemonade for \$2, how much does she spend (in \$)?

14   How many 'like' symbols are below?
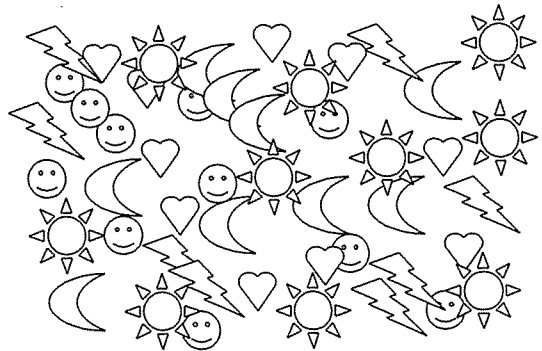
15*   A 2 L can of Colour brand paint covers $3y$ m² and a 1 L can of White brand paint covers y m². What is the total combined area covered by a 4 L can of each brand of paint?

16   Simplify $2z - z + 3y - 2y - z - y$.

17   Are there more d's or g's in the following expression?
$2d + 3x + 12d - 2x + 15g - 4g$

Collecting 'Like' Terms

'Like' terms are like identical twins.
$y \times x$, $x \times y$, xy and yx are all the same!

Name:_____

Class:_____ Due date:_____

Content	Working out	Setting out

To 'tidy up' algebraic expressions we look for, and collect, 'like' terms. Terms that are made up of two pronumerals multiplied together can be written as tg or gt, as $g \times t = t \times g$.
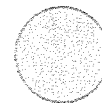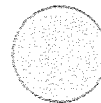
Terms such as these are best written in 'alphabetic order' (i.e. gt) so that 'like' terms are easier to see.

Consider the expression $3xy + 5x^2 - yx$.

There are three terms in this expression. The last term (yx) is usually written as xy since $y \times x$ is the same as $x \times y$.

Thus $3xy - yx = 3xy - xy$ The middle term, $5x^2 = 5 \times x \times x$
$\qquad\qquad = 2xy$

So the expression $3xy + 5x^2 - yx$ is 'tidied up' into $2xy + 5x^2$.

Example

Consider the expression $3dg + 7dg + 3g - 4gd + 4gd + 5g + d$
(a) State the number of terms.   (b) State the number of different terms.
(c) Simplify ('tidy up') the expression.   (d) How many terms are there after tidying up?

Answer

(a) There are seven terms   (b) By re-arranging the terms, we can see three
(c) $3dg + 7dg + 3g - 4gd + 4gd + 5g + d$ different types of term (dg, g and d):
$\quad = 3dg + 7dg - 4dg + 4dg + 3g + 5g + d$ $3dg + 7dg - 4dg + 4dg + 3g + 5g + d$
$\quad = 10dg + 8g + d$ (d) There are now three terms.

practice

(a) Tidy up the expression by collecting 'like' terms.
(b) State the number of terms before collecting the 'like' terms.
(c) State the number of terms after collecting the 'like' terms.

1 (a) $2x + 3y + 4x$	(b)	(c)	
2 (a) $12wx + 3yw + 14xw - 10xw + 7y$	(b)	(c)	
3 (a) $5xt + 3ty + 24x - 5tx$	(b)	(c)	

(a) Tidy up the expression by collecting 'like' terms.
(b) State the number of terms before collecting the 'like' terms.
(c) State the number of terms after collecting the 'like' terms.

4 (a) $2xt + 12y + 4xt$ (b)   (c)

5 (a) $24x + 13x + 4x - 20x$ (b)   (c)

6 (a) $12x + 3x - 15x$ (b)   (c)

7 (a) $2x + 15x + 4xg$ (b)   (c)

8 (a) $2gt - tg + 3gt + 23g$ (b)   (c)

9 (a) $12wx + wy + 4x - yw$ (b)   (c)

10 (a) $14drt - 12rdt + 5tdr + 7rgt$ (b)   (c)

11 (a) $23x + 34xy + 5x^2 - 24yw$ (b)   (c)

12 (a) $w^2 + 5w - 5w + 25$ (b)   (c)

13 (a) $y^2 + 2y + 3y + 6$ (b)   (c)

Unit 4.4

The Language of Algebra

The language of algebra allows us to convert English phrases and sentences into mathematical expressions and equations.
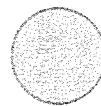
Name:_____

Class:_____ Due date:_____

Content   Working out   Setting out

The English expression 'half of a certain number' is equivalent in algebra to '$\frac{1}{2}$ of n', where n is the certain number.

The English sentence 'half of a certain number equals six' is equivalent in algebra to '$\frac{1}{2}$ of $n = 6$', where n is the certain number.

It is sometimes possible to guess and check the value of the certain number:
$\frac{1}{2}$ of 12 equals 6, so the certain number is 12.

Consecutive Numbers

1 and 2, 5 and 6, 27 and 28 are all pairs of consecutive numbers, i.e. they are next to each other.
Note that 6 = 5 + 1 (i.e. the second number in the pair) is one more than the first number.
So if a certain number is n, the next consecutive number is $n + 1$.

practice

Complete the following table where the certain number is n.

English sentence	(a) Algebraic equivalent	(b) Value of Certain No.
e.g. Half of a certain number equals six.	$\frac{1}{2}$ of $n = 6$ or $\frac{1}{2} n = 6$	12
1 Twice a certain number equals ten.		
2 Five times a certain number is ten.		
3 When a certain number is subtracted from ten, the result is six.		
4 When ten is subtracted from a certain number, the result is six.		
5 One more than twice a certain number is nine.		
6 One less than three times a certain value is fourteen.		
7 When twice a certain number is added to seven, the result is thirteen.		
8* A fifth of a certain number, then plus one equals nine.		

Set up the algebraic equations that are equivalent to the following statements.

9 Let a certain number be x. Four times the number equals twenty.	
10 Let a certain number be w. When the number is divided by six the result is eight.	
11 Let a certain number be y. When twice the number is taken away from nineteen the result is nine.	
12 Let a certain number be p. When one is added to three times the number the result is twenty-two.	
13 Let a certain number be d. The next consecutive number is twenty.	
14* Let a certain number be x. When the result of adding one to the number is multiplied by three, the answer is fifteen.	
15 Let the number of people who bought $10 tickets to the dance be n. $1450 was raised from the sale of tickets.	

16 Let the cost of an adult ticket to the play be d. A child's ticket is half price. Write the algebraic equivalent of:

(a) The cost of a child's ticket

(b) An adult and two children's tickets cost $30

(c) Two adults and five children paid $82.50 to see the play

|	
17 Let a certain number be w. When the next consecutive number is added to it, the result is 25.	
18 Let a certain number be x. When one is added to two-thirds of the number the result is seven.	

The 'Thought Box' Process

The Thought Box Process is a way of setting up an algebraic equation. Backtracking through the Thought Boxes solves the equation.

Name:_____

Class:_____ Due date:_____

Content	Working out	Setting out
◯	◯	◯

Consider the algebraic equation $4x + 1 = 21$.
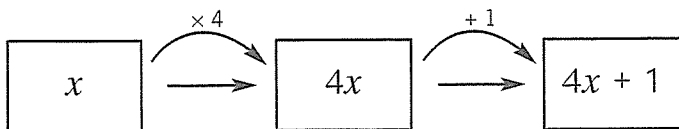
Backtracking
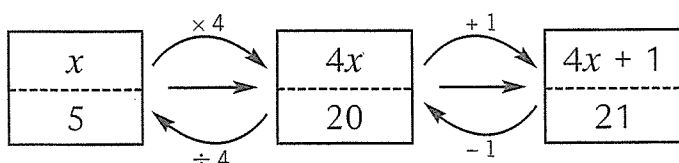
Reversing the operations, and starting at the end, instead of the beginning, solves the equation.

Thus the solution to $4x + 1 = 21$ is $x = 5$.

practice

Set up Thought Box Processes for the following equations and hence solve the equations by backtracking.

1 $3x - 4 = 20$ $x = $ ....

2 $2(x - 5) = 28$ $x = $ ....

3 $6t + 15 = 45$ $t = $ ....

Set up Thought Box Processes for the following equations and hence solve the equations by backtracking.

homework

4 $2(x - 9) = 6$

$x = $ ....

5 $\dfrac{x + 3}{5} = 4$

$x = $ ....

6 $\dfrac{2n}{3} + 1 = 7$

$n = $ ....

7 $4(y + 1) - 3 = 17$

$y = $ ....

8 $\dfrac{2(t + 3)}{5} + 7 = 11$

$t = $ ....

9 $2(3x - 4) = 16$

$x = $ ....

Expanding Brackets

If the mathematical operation does not work in arithmetic its algebraic equivalent also will not work, as arithmetic is a subset of algebra.
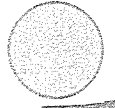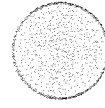
Expanding:

Multiply *every term inside the bracket* by the term outside the bracket.

Just as $3(4 + 2)$

$= 4 + 2 + 4 + 2 + 4 + 2$
$= 4 + 4 + 4 + 2 + 2 + 2$
$= 3 \times 4 + 3 \times 2$
$= 12 + 6$
$= 18$

so $3(x + 2)$

$= x + 2 + x + 2 + x + 2$
$= x + x + x + 2 + 2 + 2$
$= 3 \times x + 3 \times 2$
$= 3x + 6$

$$a \, (b + c) = a \times b + a \times c$$

Before each problem is completed, the expression should be 'tidied up' by collecting any like terms.

Example 1	Example 2
Expand $4(3x + 2)$.	Expand and simplify $6(d + 1) - 3d$.
Answer 1	**Answer 2**
$4 \, (3x + 2) = 4 \times 3x + 4 \times 2$ $= 12x + 8$	$6 \, (d + 1) - 3d = 6 \times d + 6 \times 1 - 3d$ $= 6d + 6 - 3d$ $= 3d + 6$

practice

Expand (and simplify where necessary) the following.

1	$4(x + 2)$	2	$3(4w + 9)$
3	$2(3x + 5)$	4	$\frac{1}{2}(4t + 10)$
5	$5(h + 10) + 4$	6	$3(4 + 5y) - 10$
7	$7(2x + 3) - 21$	8	$25(3 + 2y) - 50y$

Expand (and simplify where necessary) the following.

9 $12(3p + 4)$	**10** $4(3d + 7z)$
11 $6(3w + \frac{1}{2})$	**12** $10(5g + 1.7)$
13 $5(3d + 14)$	**14** $5(6w + 3) + 2$
15 $8(7 + 6m) - 56 - 48m$	**16** $6(7 + 6t) - 8t$
17 $2(5b + 6d) + 8$	**18** $4(2w + 5x) - 8w$
19 $1(y + 7) + 7$	**20*** $3(6d + 1) + 2d - 3$
21* $x(x + 4)$	**22*** $x(2x + 5) - 5x$

Unit 4.7

What is the Rule?

Algebra is a language.
A pronumeral stands in place of a number.
A rule describes the 'connection' between numbers.

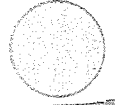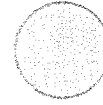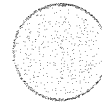Given a set of numbers, it is possible to find the connection between them.

(1)

x	y
0	1
1	2
2	3
3	4
4	5
5	6
6	7
7	8

(2)

x	y
0	0
1	3
2	6
3	9
4	12
5	15
6	18
7	21

Here are two sets of numbers that are each related by a rule (i.e. the y-value in each pair is somehow connected to the x-value).

(1) The y-value is one more than the x-value.

Algebraically $y = x + 1$.

(2) The y-value is 3 times the x-value.

Algebraically $y = 3x$.

practice

Find the rule that 'connects', each of the following sets of numbers.

1

x	y
0	−2
1	−1
2	0
3	1
4	2
5	3
6	4
7	5

2

x	y
0	5
1	6
2	7
3	8
4	9
5	10
6	11
7	12

3

x	y
0	0
1	5
2	10
3	15
4	20
5	25
6	30
7	35

4

x	y
0	0
1	0.5
2	1
3	1.5
4	2
5	2.5
6	3
7	3.5

Find the rule that connects each of the following sets of numbers.

5

x	y
0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14

6

x	y
0	1
1	3
2	5
3	7
4	9
5	11
6	13
7	15

7

x	y
0	10
1	9
2	8
3	7
4	6
5	5
6	4
7	3

8

x	y
0	15
1	14
2	13
3	12
4	11
5	10
6	9
7	8

9

x	y
0	2
1	5
2	8
3	11
4	14
5	17
6	20
7	23

10

x	y
0	9.5
1	9
2	8.5
3	8
4	7.5
5	7
6	6.5
7	6

11

x	y
0	0
1	1
2	4
3	9
4	16
5	25

12

x	y
0	1
1	2
2	5
3	10
4	17
5	26

Algebra Revision

X marks the spot, but where in the world is K2?

Name:_____

Class:_____ Due date:_____
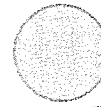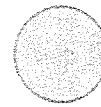
Content   Working out   Setting out

(a) Simplify the following expressions.
(b) State the number of terms (i) before
and (ii) after simplification.

1 $2x + 8y - x + 5y$	(b) (i)	(ii)
2 $23g - 3g + 4 + 4g$	(b) (i)	(ii)
3 $45t + 21 - 40t + 2 - 5t$	(b) (i)	(ii)

Evaluate the following expressions using the given conditions.

4 $3(4y - 5) + 10$ when $y = 5$	5 $\dfrac{7(5d - 4)}{3} + 3d$ when $d = 5$
6 $\dfrac{2(4t + 3)}{9}$ when $t = 6$	7 $4(c + 2) - \frac{1}{2}$ of $5c$ when $c = 4$
8 $5h + 15 - 5(h + 3)$ when $h = 2$	9 $6(h + 3g) + 4h - 3g$ when $h = 3$ and $g = 2$

Complete the following table where the certain number is n:

English sentence	(a) Algebraic equivalent	(b) Value of Certain No.
10 Adding three to twice a certain number results in fifteen.		
11 Half a certain number equals ten.		

Set up Thought Box Processes for the following equations and hence solve the equations by backtracking.

12 $4x - 5 = 23$

$x = $ ....

13 $3(x + 7) = 33$

$x = $ ....

14 $5t + 15 = 65$

$t = $ ....

Expand (and simplify where necessary) the following.

15 $14(3x + 2)$	16 $4(3w + 7) - 28$
17 $4(3x + 11) - 12x$	18 $2(3x + 2y) + 4$
19 $16 + 3(2x + 5)$	20* $\frac{1}{2}(4t + 10) + \frac{1}{2}(4t + 10)$